Chaos at the limits of computation

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In 1972, P.W. Anderson coined the phrase “More is Different" to describe a philosophical revolution in physics: that the emergent phenomena displayed by a physical system with many individual constituents could differ from what can be derived by considering its constituents. But physics is a theory grounded on empirical observation. Could the limitations of computationally bounded observers represent another complexity scale for physics?

We study quantum chaos at the limit of bounded computation. Chaos is a quantum many-body phenomenon associated with a number of intricate properties, such as level repulsion in energy spectra or distinct scalings of out-of-time ordered correlation functions. These form the basis for how chaos in a candidate system is often diagnosed: by checking for these emergent properties in the Hamiltonian governing the system. This “checklist" approach is far from foolproof: it has been found that non-chaotic systems can display some of these phenomena too. We show the converse: a system displaying none of these phenomena could nevertheless look chaotic to every bounded observer.

We do this by spoofing the Gaussian Unitary Ensemble (GUE), which is often taken as a quintessential model of random Hamiltonians whose time evolution is chaotic. We introduce a novel class of “pseudoGUE” quantum Hamiltonians that demonstrate Poissonian level statistics, low operator complexity, and weak scrambling properties – thus failing every item on the chaos checklist – yet are indistinguishable from true GUE Hamiltonians by any bounded-time experiment or computation. As an application, we give an efficient quantum algorithm to simulate Hamiltonians from our ensemble, thus bypassing a no-go theorem stating that true GUE Hamiltonian evolution can only be simulated in exponential time. Our work establishes fundamental limitations on Hamiltonian learning and testing protocols and derives stronger bounds on entanglement and magic state distillation.