Quantum Fourier transform beyond Shor's algorithm: Exercise Sheet 4

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András Gilyén (Alfréd Rényi Institute of Mathematics)

Feel free to skip exercises that you find too easy or hard. On the last page you can find some hints where indicated by (**H**).

Exercises

1.) A Markov chain can be described as a walk on a graph. What does the detailed balance condition

$$P_{ji}\pi_i = P_{ij}\pi_j$$

tell about the transfer of "probability mass" along the edges? Prove that if the Markov chain P satisfies the above detailed balance condition with respect to the distribution π, then π is a fixed point, i.e., $P\pi = \pi$.

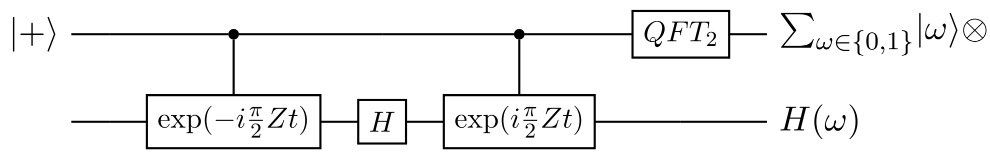
2.) Prove that the Metropolis rule applied to a symmetric Markov chain $P = P^T$:

- if $\tau_{z'} \geq \tau_z$ accept the move,
- if $\tau_{z'} < \tau_z$ reject the move with probability $1 - \frac{\tau_{z'}}{\tau_z}$,

ensures that the modified Markov chain $P^{(\tau)}$ is detailed balanced with respect to τ.

3.) Based on the above how would you define detailed balance for infinitesimal generators L (Laplacians)? What does detailed balance imply about $L\pi$, and $e^{tL}\pi$?

4.) Compute the operator Fourier transform of the operator $K \leftarrow H$, with respect to the Hamiltonian $H \leftarrow \frac{\pi}{2}Z$ with F_2 as the discrete Fourier transform. Compute $H(0)$ and $H(1)$. Verify that $(H(0))^\dagger \cdot H(0) + (H(1))^\dagger \cdot H(1) = I$.



5.) (**H**) Decompose a unitary U according to the Bohr frequencies Δ of any Hamiltonian H. Show that $\sum_{\Delta \in B}(U^{(\Delta)})^\dagger \cdot U^{(\Delta)} = I$.

Hints

Exercise 5: First consider the case when all Bohr-frequency corresponds to a single pair of eigenstates. Then study what happens when you "group" several pairs of eigenstates into a Bohr frequency Δ.